NUMERICAL SIMULATION OF CAVERN SHAPE DEVELOPMENT DURING PRODUCT WITHDRAWAL WITH FRESH WATER – THEORY AND CASE STUDY

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ABSTRACT

Salt caverns are frequently used for storing large quantities of liquid hydrocarbons. If these products are withdrawn by unsaturated brine or fresh water, the size and shape of the cavern will change: These changes are sometimes significant, particularly if several turnovers have been performed. Most of the computer programs for modelling leaching processes are not able to simulate these changes as they do not cope with moving blanket/product levels. The computer program PROSACAV 4.0 however allows for simulation of leaching processes with gradually changing blanket levels and therefore product injection and withdrawal processes. This paper will compare the results obtained by the computer code with an analytical solution for re-leaching processes in a cylindrical cavern. Following PROSACAV 4.0 is applied to simulate the cavern shape and size development of a cavern with unfavourable starting contour by multiple product turnovers with fresh water and in combination with regular leaching operations.

Introduction

Storage caverns in salt for liquid products are subject to significant volume and shape changes if fresh water or low-salinity brine is used to withdraw the storage product. That phenomenon is well known and the major effects can be summarized in general in 2 theses:

1. Volume increase is of the order of 17 % per turn-over (if fresh water is used for product withdrawal).

2. Cavern shape changes are more distinctive in the lower than in the upper part of the cavern

Volume increase and the change in cavern shape are the result of leaching processes induced by the injected fresh water when coming into contact with the rock salt. Consequently, formulas and procedural methods typically used in leaching simulations have to be applied in order to quantify those effects. But there is an aspect that makes an important difference between salt cavern leaching for the purpose of solution mining on the one hand and product withdrawal on the other hand: Typically the leaching process is characterized by a constant-in-depth blanket level during leaching stages, whereas product discharge processes are featured by a continuously elevating "blanket" level represented by the storage product-brine interface.

Typically, leaching simulation programs have been developed in order to model standard solution mining, i. e. processes characterized by a constant blanket level during a leaching stage. They are normally not capable of coping with a gradually changing blanket level and therefore not suited to simulate leaching during product discharge with sufficient accuracy.

Calculation of leaching effects in the vicinity of gradually changing blanket-brine interface is not that simple because several mutually influencing and interfering phenomena coincide:

- Elevation of the blanket brine interface upwards which corresponds to a permanent change of the calculation area
- Propagation of the cavern contour due to dissolution which results also in a permanent change of the calculation area, but in a perpendicular direction
- Transient change of brine mineralization influencing the dissolving rate at the contour

In order to make that complex, time-dependant process accessible to numerical computation, it must be decoupled into several sub-processes which are to be treated separately and subsequently in the following.

PROSACAV 4.0 (PSC 4) is a computer code which simulates 3D leaching processes under complicated geological conditions. It was developed to also model salt dissolution processes during product withdrawal from salt caverns.

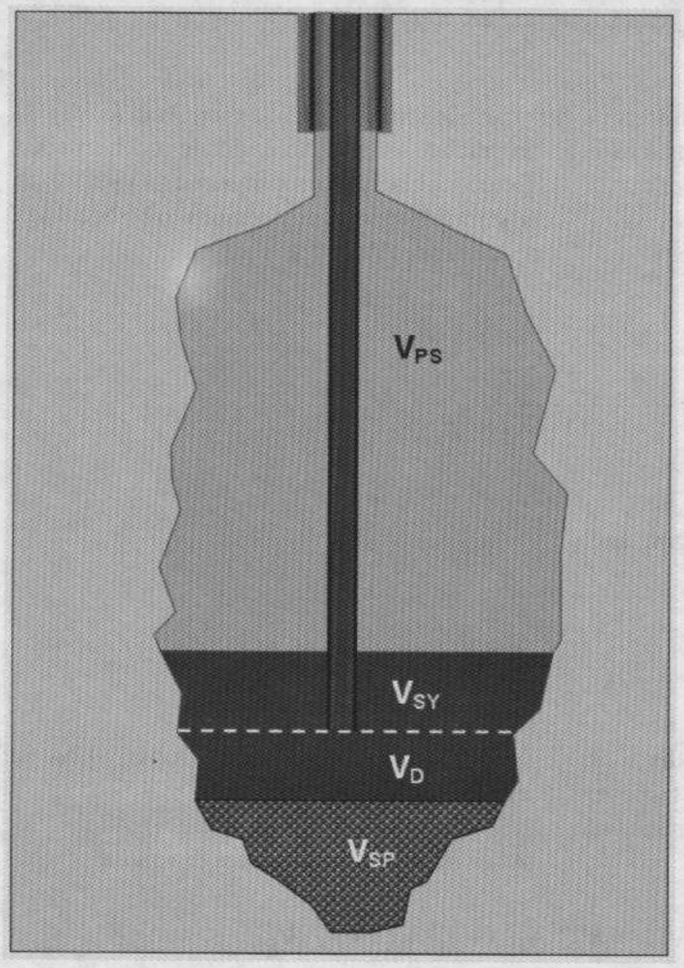
The question is whether the algorithms used in the program are reliable and accurate enough for practical purposes. In order to evaluate the accuracy of the computer code the comparison of numerical results with field test data from sonar surveys is in general possible. But this requires high expenses for string manipulations and precise sonar surveys before and after turnover.

Usually, the mathematical description of transient leaching processes by analytical solutions causes difficulties. Fortunately, a simplified analytical formula for the description of re-leaching effects in a cylindrical cavern exists [1]. That formula can be used to benchmark the accuracy of numerical results obtained by PSC 4.

The paper at hand therefore

- gives a general introduction to the calculation of re-leaching effects
- reminds of the analytical solution for re-leaching a cylindrical cavern
- explains the procedural methods used in PSC 4 to simulate the leaching process while blanket level move upwards or downwards
- compares the results of the analytical solution with those of the computer code
- demonstrates the application of PROSACAV 4.0 to a cavern with unfavourable starting shape that is intended to be used for both liquid product storage and brine production.

Definition of Terms in Re-leaching



The picture shows the simplified scheme of a typical liquid product storage cavern. Specific well completion equipment like e. g. water injection string inside the brine string is omitted for simplicity reasons.

V_{PS}	Volume of the liquid Product Stored
V_{SY}	Safety volume to avoid overfilling
V_D	Dead volume to avoid suction of insoluble material into the brine string
V_{SP}	Sump volume
ΔV_{salt}	amount of pure salt dissolved per turnover
a	Volume content of insoluble material in rock salt
b	bulk factor for insoluble material in the sump

Increase in Cavern Volume

The following amount of salt is dissolved when the product stored was completely withdrawn by unsaturated brine or fresh water with mass concentration Z_0 and density ρ_0 :

Formula 1

$$\Delta V_{salt} = \frac{Z_S - Z_0}{1 - Z_S} \cdot \frac{\rho_0}{\rho_{salt}} \cdot V_{PS}$$

The salt dissolution affects not only the storage volume V_{PS} but also the safety volume V_{SY}. If the rock salt exhibits a significant content of insoluble material (volume share a) and if the dead volume V_D is large enough to accommodate the released insoluble parts, then the increase of storage and safety volume together is larger than stated in Formula 1:

Formula 2

$$\Delta V_{PS} + \Delta V_{SY} = \frac{1}{1-a} \cdot \Delta V_{salt}$$

As will be shown in the following, the safety volume increase V_{SY} is different from that of the storage volume V_{PS}. Apportioning of that volume increase to V_{SY} and V_{PS} depends on a number of aspects and is difficult to be quantified in a simple manner. And, as V_{SY} is in practice usually dimensioned much smaller for economic reasons compared to V_{PS}, V_{SY} is assumed for simplicity reasons to be equal to 0. In that case ($V_{SY} = 0$)

. 2 takes the form:

Formula 3

$$\Delta V_{PS} = \frac{1}{1-a} \cdot \frac{Z_S - Z_0}{1 - Z_S} \cdot \frac{\rho_0}{\rho_{salt}} \cdot V_{PS}$$

For fresh water at 20 °C ($Z_0 = 0$ and $\rho_0 =$ 998.3 kg/m³) and halite ($\rho_{salt} = 2164$ kg/m³ and $Z_S = 0.2645$)

. 3 takes the form:

$$\Delta V_{PS} = \frac{1}{1-a} \cdot 0.166 \cdot V_{PS}$$

If there is no insoluble material in the rock formation, the volume increase per turnover amounts to ~ 16.6 %. If the content of insoluble rock material however is 10 %, the increase in volume goes up to 18.4 %.

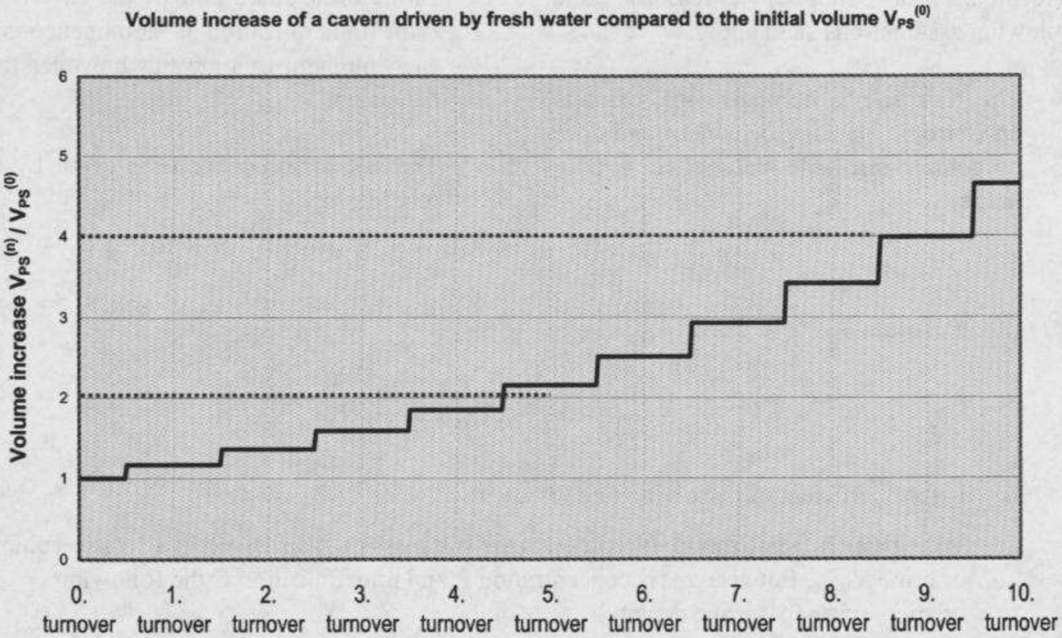
After first turnover the storage volume reaches

$$V_{PS}^{(1)} = V_{PS}^{(0)} + \Delta V_{PS} = V_{PS}^{(0)} + f \cdot V_{PS}^{(0)} = (1+f) \cdot V_{PS}^{(0)}$$

$$\text{with } f = \frac{1}{1-a} \cdot \frac{Z_S - Z_0}{1 - Z_S} \cdot \frac{\rho_0}{\rho_{salt}}$$

and, finally after n turnovers:

$$V_{PS}^{(n)} = (1+f) \cdot V_{PS}^{(n-1)} = (1+f)^n \cdot V_{PS}^{(0)}$$



From the diagram it can be seen that after 5 turnovers, the storage volume of a cavern in pure halite is doubled, after 9 turnovers it is quadrupled (provided that convergence is negligible). If the content of insoluble material is significant then the reduction of the dead volume by accumulation in the sump has to be taken into account:

$$\Delta V_D = -\frac{a \cdot b}{1-a} \cdot \frac{Z_S - Z_0}{1 - Z_S} \cdot \frac{\rho_0}{\rho_{salt}} \cdot V_{PS}$$

Example: The initial storage volume amounts to 500,000 m³, the dead volume is assumed to be 5 % of the storage volume (25,000 m³). Content of insoluble material is 5 %, bulk factor = 1.5.

Development of storage and dead volume is as follows:

turnover	storage volume V_{PS}	dead volume V_D
0	500,000 m³	25,000 m³
1	587,368 m³	18,447 m³
2	690,003 m³	10,750 m³
3	810,572 m³	1,707 m³

It can be seen that after merely 3 turnovers the dead volume of 5 % of the initial storage volume is nearly completely exhausted and pulling of the water injection string becomes necessary.

Re-leaching of a Cylindrical Cavern

The relations derived thus far are solely quantitative and do not contain any information about the distribution of re-leaching effects versus cavern height, i. e. the change of cavern shape. Below a formula for calculating the re-leaching effects in a cylindrical cavern based on [1] shall be presented. For simplicity reasons, the following assumptions shall apply:

- The rock salt is homogeneous, exhibits an isotropic leaching behaviour and does no contain insoluble material (e. g. pure halite)
- In the brine filled part of the cavern the brine mineralization is homogeneous in each moment of time but however time dependent
- The salt dissolution rate is given by the relationship

Formula 4

$$\frac{d\,m_{salt}}{d\,t} = K \cdot A \cdot \left(C_S - C(t)\right)$$

K is the coefficient of dissolving rate for a vertical wall,

C is the brine mineralization. It represents the salt mass m_{salt} dissolved in a certain volume of brine V_{brine}. Between mass concentration Z and mineralization C the following relation exists (ρ – brine density):

$$Z = \frac{C}{\rho}$$

C_S is the mineralization at saturation point

A is the surface from which the salt is dissolved

- After complete removal of the storage product, the cavern roof remains covered by a small layer of product, thus the roof does not contribute to brine saturation.

The entire leaching process has to be split in 2 phases:

- The product withdrawal phase
  In this phase the volume of brine, its mineralization and the active leaching surface change in time because the brine-product interface moves upwards. The velocity of raising the interface level depends on the flow rate Q and the cavern diameter R.
- The saturation phase
  In this phase the cavity is (nearly) completely filled with brine striving to reach the saturation point. The dissolving area and the brine volume are assumed to be constant.

For each of theses phases a salt mass balance is set up which is used to obtain a differential equation describing the mineralization development versus time. After insertion of the resulting expression C(t) into Formula 4 the integration yields a formula describing the total radius increase versus cavern height R(h) during the 1st turnover (see attachment 2 and 3).

It has to be stressed that in these calculations the safety V_{SY} volume plays an important role and must necessarily be taken into account.

The Product Withdrawal Phase

The brine in the safety volume at the beginning of the process is assumed to be saturated. Then the mineralization development versus time is given by:

Formula 5

$$C_{(1)}(t, t \le V_{PS}/Q) = C_S - (C_S - C_0) \cdot \frac{R}{2 \cdot K \cdot (t + V_{SY}/Q)} \cdot \left(1 - \exp\left(-\frac{2 \cdot K \cdot t}{R}\right)\right)$$

The resulting radius increase $\Delta R_1(h)$ in this phase is:

Formula 6

$$\Delta R_1(h) = \frac{R \cdot (C_S - C_0)}{2 \cdot \rho_{salt}} \cdot \left\{ \ln\left(\frac{V_H}{V_h}\right) - \exp\left(\frac{2 \cdot K \cdot V_{SY}}{Q \cdot R}\right) \cdot \left[Ei\left(-\frac{2 \cdot K}{R} \cdot V(H)\right) - Ei\left(-\frac{2 \cdot K}{R} \cdot V(h)\right)\right]\right\}$$

whereas Ei(x) is the integral-exponential function:

$$Ei(x) = D + \ln(-x) + \sum_{i=1}^{\infty} \frac{x^i}{i \cdot i!}, \; x < 0$$

$$V(H) = V_{SY} + \pi \cdot R^2 \cdot H$$

$$V(h) = V_{SY} + \pi \cdot R^2 \cdot h$$

The Brine Saturation Phase

The brine mineralization at the beginning of the saturation phase is given by Formula 5 with $t_1 = V_{PS}/Q$.

Formula 7

$$C_{(2)}(t, t > V_{PS}/Q) = C_S - (C_S - C_0) \cdot \frac{Q \cdot R}{2 \cdot K \cdot (V_{PS} + V_{SY})} \cdot \left(\exp\left(\frac{2 \cdot K \cdot V_{PS}}{Q \cdot R}\right) - 1\right) \cdot \exp\left(-\frac{2 \cdot K \cdot t}{R}\right)$$

In this phase radius increase is independent on the height and amounts from:

Formula 8

$$\Delta R_2(t > t_1) = \frac{R \cdot (C_S - C_0)}{2 \cdot \rho_{salt}} \cdot \frac{Q \cdot R}{2 \cdot K \cdot (V_{PS} + V_{SY})} \cdot \left(1 - \exp\left(-\frac{2 \cdot K \cdot V_{PS}}{Q \cdot R}\right)\right)$$

The Final Solution

The brine mineralization in the cavern versus time:

$$C_{(1)}(t, t \le V_{PS}/Q) = C_S - (C_S - C_0)\cdot \frac{R}{2\cdot K\cdot(t+V_{SY}/Q)}\cdot\left(1-\exp\left(-\frac{2\cdot K\cdot t}{R}\right)\right)$$

$$C_{(2)}(t,t>V_{PS}/Q) = C_S-(C_S-C_0)\cdot\frac{Q\cdot R}{2\cdot K\cdot(V_{PS}+V_{SY})}\cdot\left(\exp\left(\frac{2\cdot K\cdot V_{PS}}{Q\cdot R}\right)-1\right)\cdot\exp\left(-\frac{2\cdot K\cdot t}{R}\right)$$

The total cavern radius increase $\Delta R(h)$ when brine saturation was reached is the sum of the terms given by Formula 6 and Formula 8:

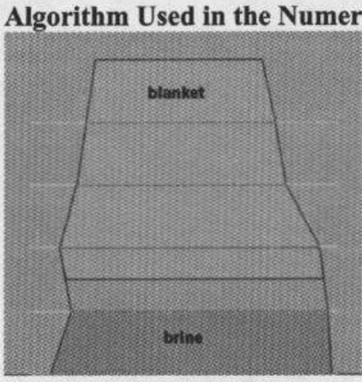
$$\Delta R = \Delta R_1(h) + \Delta R_2$$

It has to be emphasized that the analytical solution is merely a simplified approximation too. The deduction of the formulas is based on the assumption that during the whole re-leaching process the cylindrical shape is maintained. Neither volume increase and cavern shape changes nor the influence of changing cavern wall inclination on the dissolution rate are taken into account.

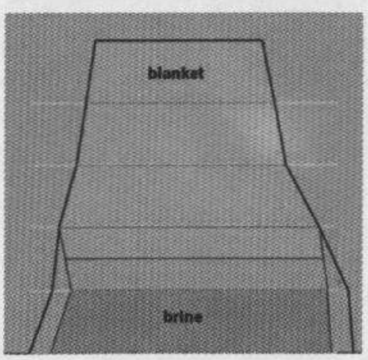
The calculations in PSC 4 are based on a layered structure of the cavern shape and discrete time steps Δt. Within such time step the product-brine level goes upwards and can pass through even more than 1 layer if the layer structure is subtle enough. During that time step the mineralization and the contour movement between the former and the new product brine level is undefined.

Algorithm Used in the Numerical Model

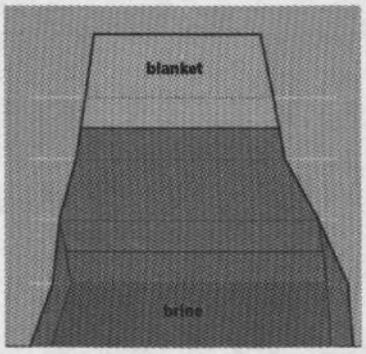
In order to make that problem accessible to calculation the following approach was chosen:



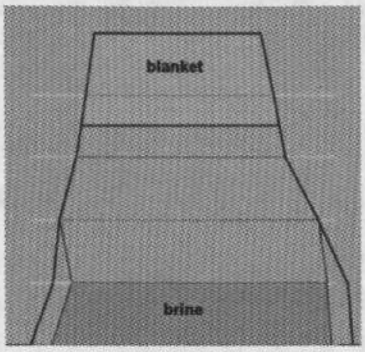
1. Situation in the cavern at $t = t_0$.



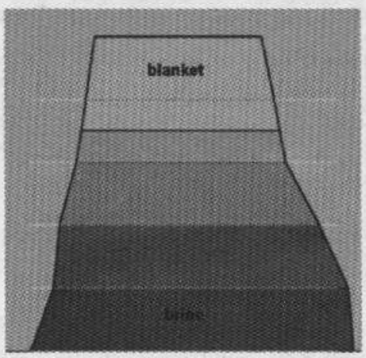
2. Calculation of contour progression within Δt with the blanket/product level as at $t = t_0$ (initial blanket/product level).



3. Determination of the depth of the new blanket/product level with the new contour at time $t = t_0 + \Delta t$



4. The cavern layers successively filled during Δt with brine are assumed to exhibit at $t = t_0$ a mineralization identical to that of the previously most upper brine-filled layer.



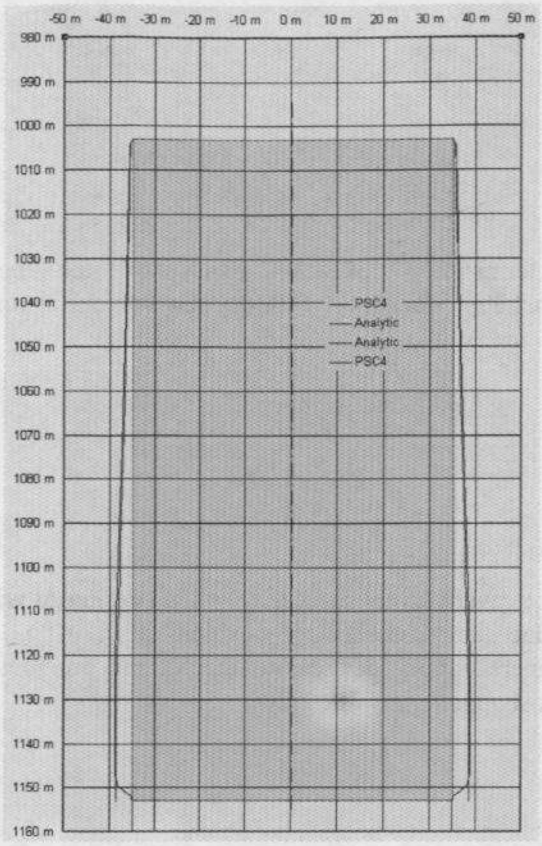
5. Calculation of mineralization change in the newly brine-filled layers as if they had been filled completely with brine during the entire time of step Δt .

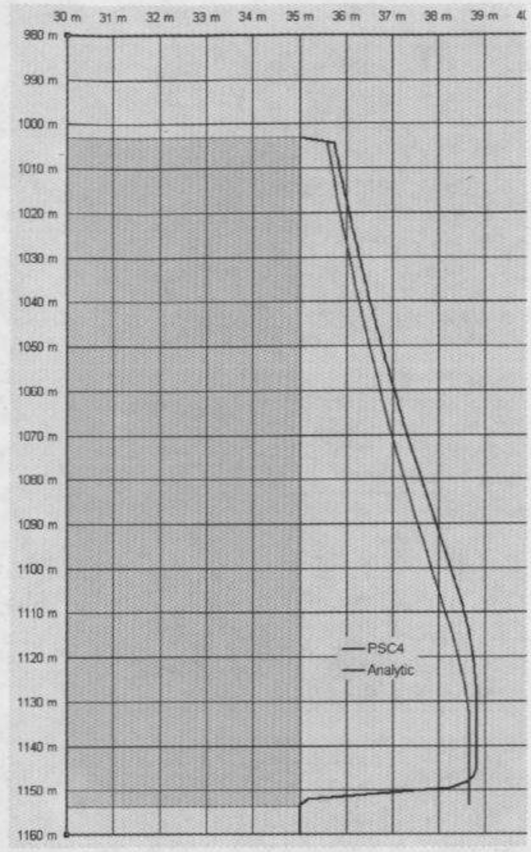
Comparison of the Analytical Solution with Numerical Results

Case 1 – 500,000 m³ Cavern Depleted with 300 m³/h

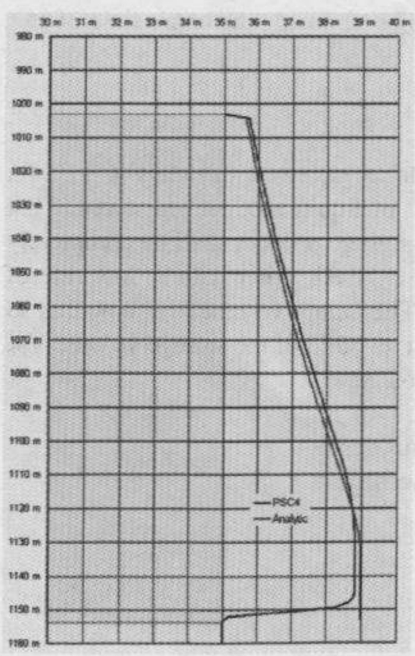
In a cylindrical cavern with radius $R = 35$ m, 500,000 m³ of liquid product are stored. That amount of storage product shall be completely withdrawn by fresh water with a flow rate of 300 m³/h. The safety volume V_{SY} amounts to 75,000 m, i. e. 15 % of the storage volume V_{PS}. The question is:

"Which cavern radius increase versus depth has to be expected after product discharge?"





The computational results drawn on the scale exhibit a quite good conformity of both curves. A closer look at the curves in a not-to-scale presentation shows on the one hand a good similarity in contour characteristics but reveals on the other hand some deviations in absolute values.



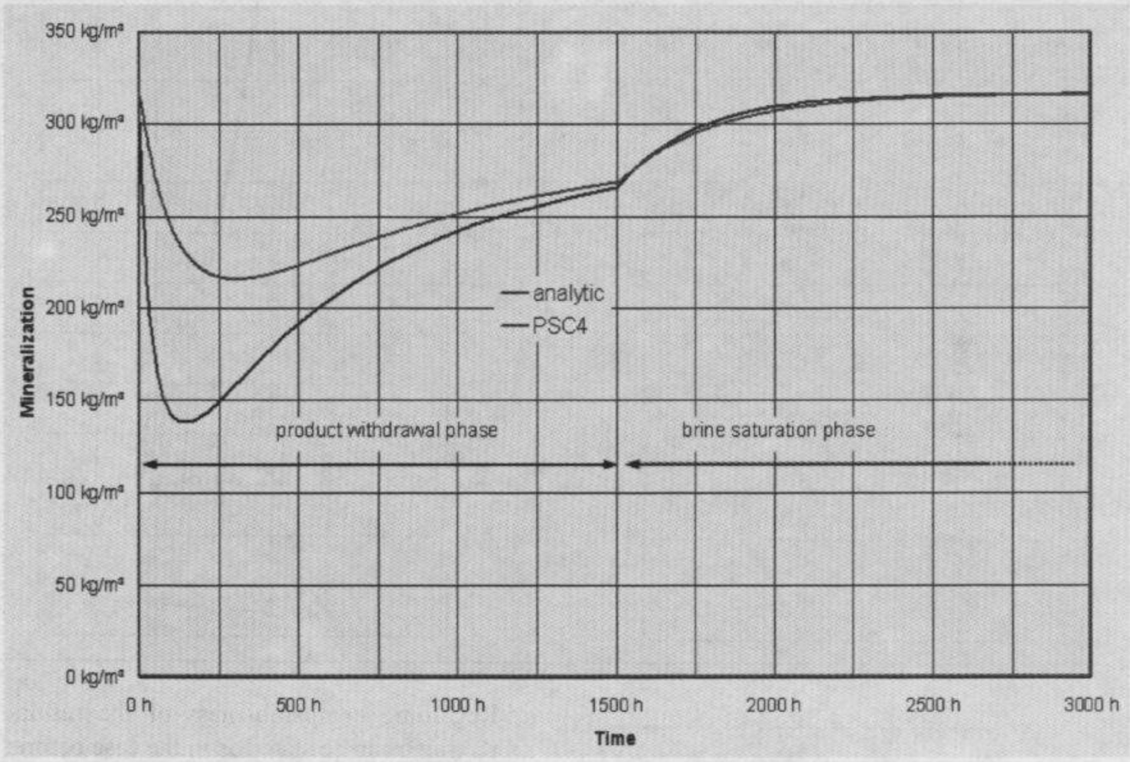
Comparison of the numerical results with corrected radius increments of the analytical solution

The radius increase according to the analytical solution is smaller compared to that of the numerical computation.

This is however plausible because the analytical solution does not take into account – as already mentioned - the salt contained in the cavern volume additionally created by re-leaching. This explanation is in accordance with the figures for the volume increase: The contour change based on the analytical solution corresponds to a volume increase of only 15.2 % (pertaining to the storage volume), whereas the numerical calculations yield an increase of 16.6 %, which is in compliance with the theoretical value according Formula 1.

If a correction factor of f = 1.092 is introduced to multiply the radius increment of the analytical solution, an identical volume increase of 16.6 % is obtained and a much better compliance of both curves can be observed.

The mineralization development versus time also exhibits some deviations, particularly in the initial phase of product discharge:

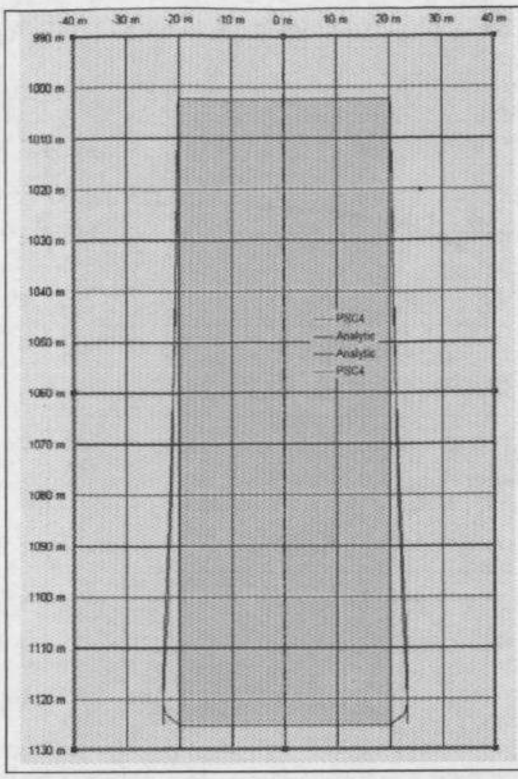


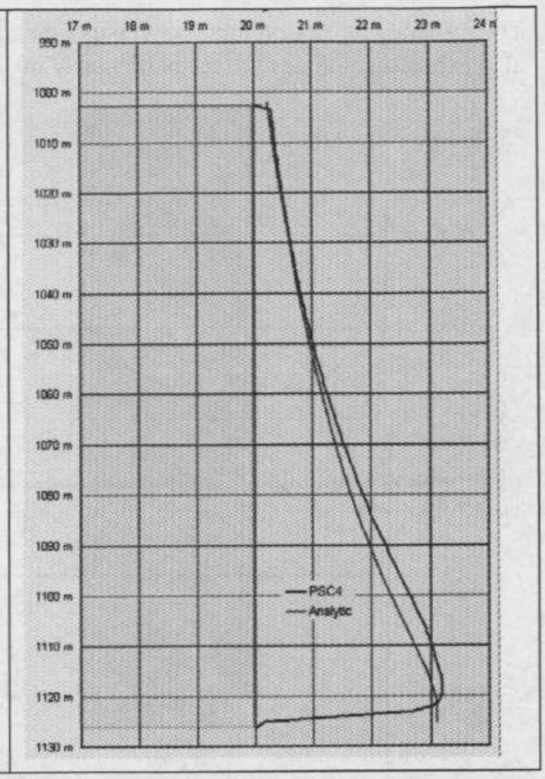
This may have also been determined by the simplifying assumption of the analytical solution that the cylindrical contour remains constant. The numerical calculation however takes into account the increasing surface in the lower cavern part and consequently the intensification of salt dissolution process resulting in a higher mineralization.

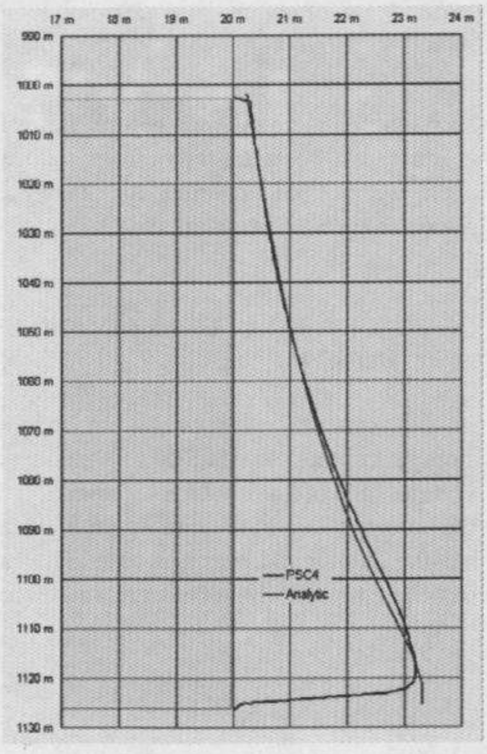
At the end of the product withdrawal phase, the difference is rather minor. From the diagram it can be seen that the brine mineralization in the cavern after product discharge is still far from the saturation point. A time estimation can therefore be made for how long one should wait before the residual product can be withdrawn from the cavern neck without endangering the casing shoe region from dissolution effects.

Case 2 – 150,000 m³ Cavern Depleted with 100 m³/h

In this case the storage cavern exhibits a storage volume of 150,000 m³ and a safety volume of 5,000 m³, which is approx. 3 % of the storage volume and therefore much smaller than in the previous case. The cavern shall be emptied with a flow rate of 100 m³/h.







Comparison of the numerical results with corrected radius increment values of the analytical solution

Regarding comparableness of the results, the same can be stated as in the case before:

- Radius increment obtained from the analytical solution is generally smaller compared to the numerical results. The same relates to the cavern volume increase (+15.3 % according the analytical solution, +16.6 % of the numerical results)
- If the numerical results are proportionally corrected, the obtained conformity of the curves is much better. In that case the deviation between both curves at depth 1,100 m is approx. only 7 %.

Both cases illustrate that the quantity of the safety volume and the product withdrawal rate significantly influence the contour shape development. The same pertains to the starting mineralization of the water/brine used to discharge the product. A more detailed investigation in this regard is shown in [2].

Summary of the Comparison

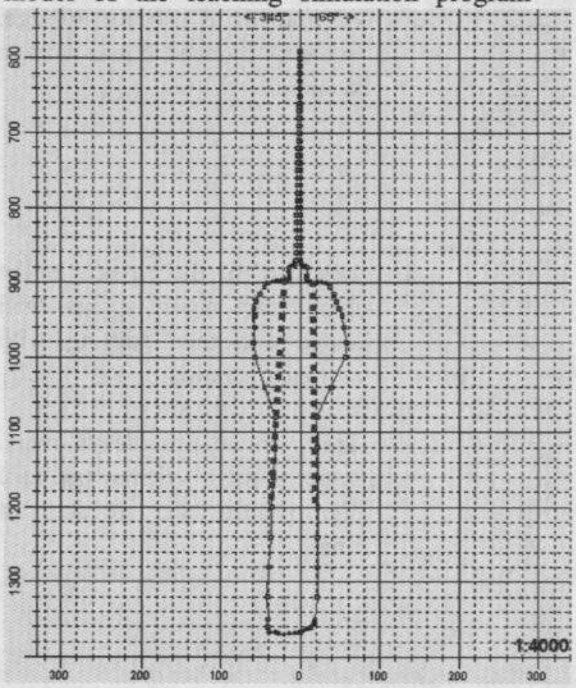
The plots regarding brine mineralization and cavern shape development obtained by the analytical solution and the numerical computation exhibit a good similarity in terms of curve characteristics. A more detailed analysis of the figures however reveals some deviations. These deviations can be plausibly explained by the simplifying assumptions which the analytical solution is based on. The volume increase determined by the numerical model is in full compliance with the theoretical value.

It can therefore be stated that the numerical model of the leaching simulation program PROSACAV 4.0 describes the leaching effects during continuously moving blanket level correctly and with sufficient accuracy for practical demands.

Case Study

The Problem

During leaching of a large brine production cavern in direct leaching mode untightness in the casing shoe region occurred and resulted in leakage of the liquid blanket.



This leakage has remained undiscovered over a certain period of time. Consequently, the cavern neck over a length of more than 300 m was significantly widened.

Afterwards, a cavern sonar survey was performed and revealed an even worse situation: Presumably caused by damage to the water injection string, the cavern exhibited a bellied extension in the middle part with maximum diameter of already 103 m.

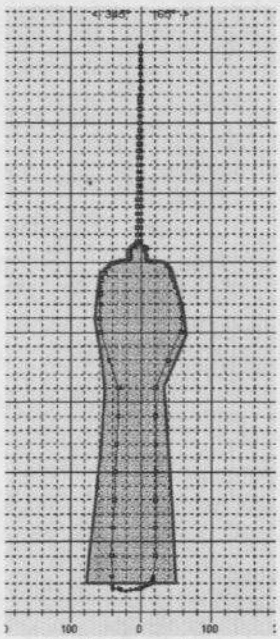
The cavern operator intends to plug and abandon the un-tight well and to drill a new well into the existing cavern. In that context he was interested to learn:

- How much salt can still be produced from that cavern while keeping the distances to adjacent caverns within the required limits (pillar width)?

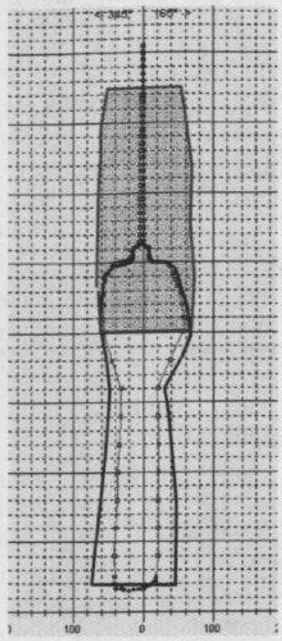
  In which way can this cavern be used for crude oil storage for strategic purposes and how will the cavern volume and shape develop?

In order to answer these questions, two leaching and storage scenarios have been developed. The computer program PROSCAV 4.0 was used to simulate the corresponding leaching and storage processes.

Leaching and Storage Concepts



First: Operation

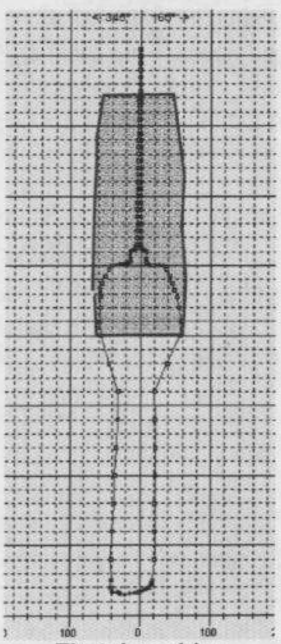


Then: Leaching

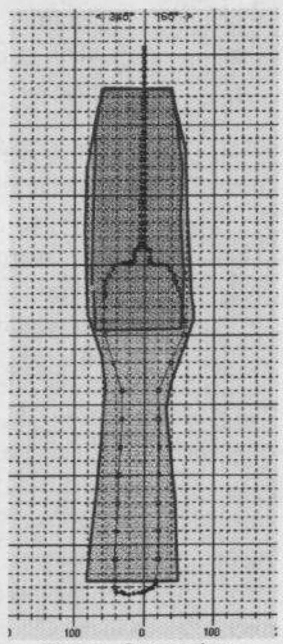
Two different strategies to solve the problem have been identified:

1. Immediate use of the cavern for storage purpose. After reaching the critical diameter in the lower part either the storage depth interval will be shifted upwards or the cavern will be leached in the upper part to its final size (or combination of it)

   That version has the advantage of immediate use for storage, but with reduced initial storage volume.



First: Leaching



Then: Operation

2. Leaching of the cavern in the upper part nearly to its final shape and then starting with storage operation.

   This version provides a much larger storage volume but at a significantly later moment of time.

Calculations and Results

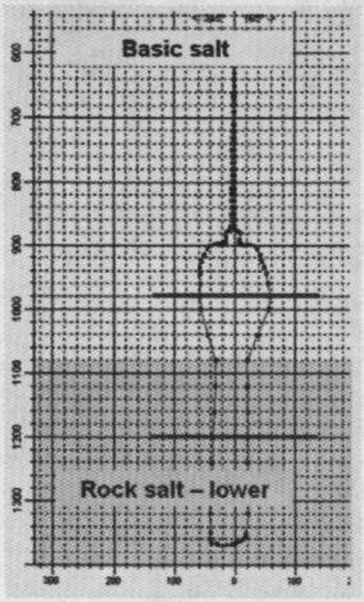
The initial cavern net volume amounts to nearly 1,900,000 m3. The content of insoluble material is small, on average 2 % only.

The bulk factor for the insoluble material is assumed to be 1.3. Hence ~ 3 % of the cavern volume will be occupied by the sump.

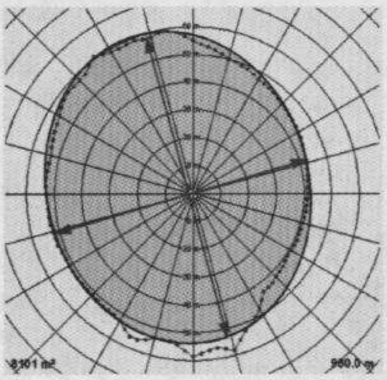
Terms of references:

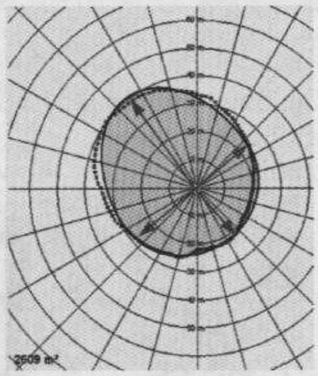
- Maximum diameter of 125 m is permissible (deduced from the maximum horizontal cross section measured). This corresponds mathematically to a circular cross section area of 12,272 m².
- The pillar thickness to the adjacent caverns must not be smaller than 125 m.
- The brine produced shall exhibit a salt mineralization of not less than 312 kg/m³.
- The possibility of brine production has to be investigated for fresh water flow rate of 150 m³/h.
- The possibility of oil storage has to be investigated for product withdrawal rate of 500 m³/h and 1,000 m³/h:

Salt leaching behaviour:



From the sonar survey an anisotropic leaching behaviour of the salt was deduced. The salt formation in the cavern depth interval was therfore divided in two parts each with different properties which depend on azimuth of the leaching direction.

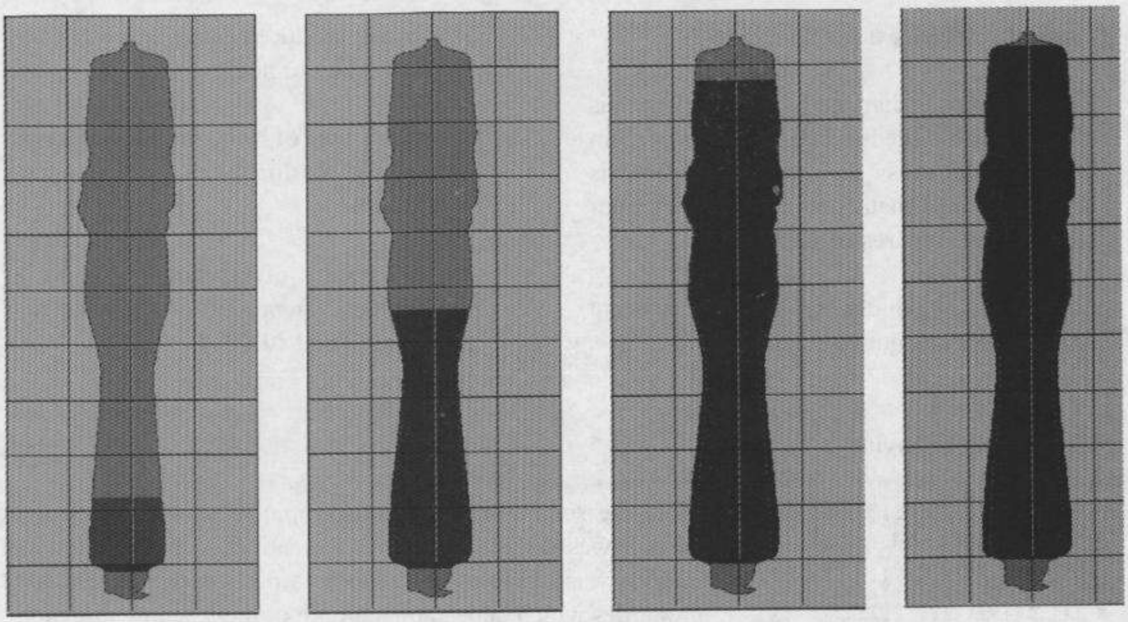




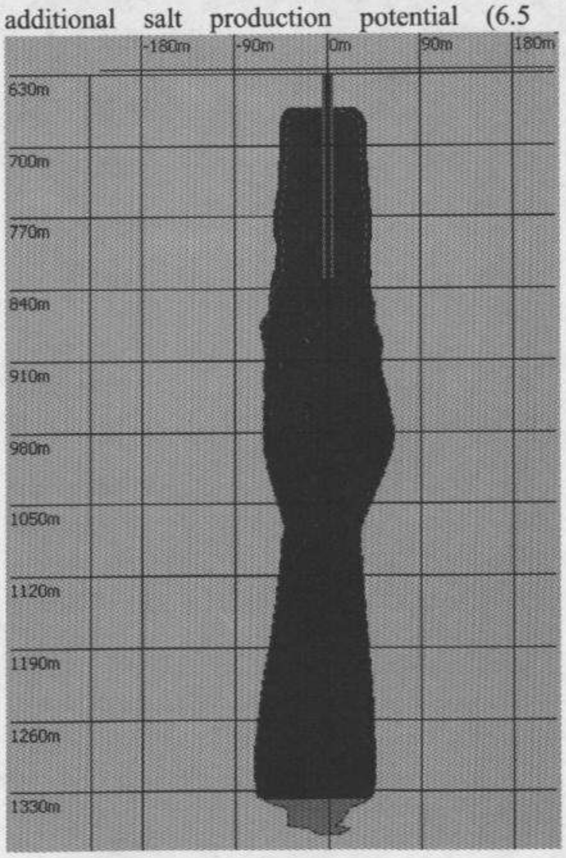
The results

The following pictures illustrate the gradual change in brine mineralization during product withdrawal. The intensity of the blue colour is a scale for the brine mineralisation. It is visible that at the very beginning of product discharge the brine mineralization is low. With rising product-brine level the brine becomes more concentrated.

The white dotted line represents the cavern shape at the beginning of the product withdrawal process.



Regarding the computational results, both options are comparable in terms of final cavern volume (~ 5,650,000 m³) and additional salt production potential (6.5 million Mg). Major differences – as initially expected – emerge in the storage potential.



Example of the final cavern shape according option "First storage, then leaching"

Option "First storage, then leaching"

- 5 storage cyles are possible with storage volumes ranging from 1,900,000 m³ to 2,600,000 m³
- Following leaching in the upper part over 9 years with 150 m3/h
- Maximum horizontal cross section calculated:11,091 m² (12,272 m² permissible)

Option "First leaching, then storage "

- Leaching with 150 m³/h over 7 years up to net volume 3,300,000 m³
- 4 storage cyles are possible with storage volumes ranging from 3,100,000 m³ to 4,400,000 m³
- Maximum horizontal cross section calculated: 12,578 m² (12,272 m² permissible)

Conclusions

The leaching process in the vicinity of a gradually moving blanket-brine interface is a quite complex process. By de-coupling of that process into several separate sub-processes, it is possible to simulate this process numerically. The comparison of the numerical results with an analytical solution for re-leaching a cylindrical cavern revealed a satisfying compliance.

The computer program PROSACAV 4.0 based on the presented algorithm for calculation of cavern contour changes and mineralization development below the blanket brine interface therefore allows for the simulation of re-leaching processes during product withdrawal with a gradually upwards moving product-brine interface.

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Attachments

Attachment 1 – Calculation of volume increase by salt dissolution

Mass concentratration of the brine/water in the volume V_{PS} before salt dissolution:

$$Z_0 = \frac{m_{salt}}{m_{salt} + m_{H_2O}} = \frac{m_{salt}}{\rho_0 \cdot V_{PS}}$$

At the saturation point the water mass remains the same, but an additional salt mass $\Delta m_{salt} = \rho_{salt} \cdot \Delta V_{salt}$ has been dissolved:

$$Z_S = \frac{m_{salt} + \Delta m_{salt}}{m_{salt} + \Delta m_{salt} + m_{H_2O}} = \frac{m_{salt} + \rho_{salt} \cdot \Delta V_{salt}}{\rho_0 \cdot V_{PS} + \rho_{salt} \cdot \Delta V_{salt}}$$

Comparison of both terms yields:

$$\boxed{\Delta V_{salt} = \frac{Z_S - Z_0}{1 - Z_S} \cdot \frac{\rho_0}{\rho_{salt}} \cdot V_{PS}}$$

Attachment 2 –Calculation of the product withdrawal phase

$$C(t+\Delta t) \cdot V(t+\Delta t) = C(t) \cdot V(t) + K \cdot A(t) \cdot \left(C_S - C(t)\right) \cdot \Delta t + C_0 \cdot Q \cdot \Delta t$$

with

$C(t+\Delta t)\cdot V(t+\Delta t)$	-	salt mass in the cavern at t + Δt
$C(t)\cdot V(t)$	-	salt mass in the cavern at t
$K\cdot A(t)\cdot\big(C_S - C(t)\big)\cdot\Delta t$	-	salt dissolved during Δt from the cavern surface with area A(t)
$C_0\cdot Q\cdot\Delta t$	-	salt conveyed with the water/brine injected into the cavern
$\dfrac{dV}{dt}=Q$	-	Change of brine volume versus time
$V(t)=Q\cdot t+V_{SY}$	-	Brine filled volume at time t
$A(t)=\dfrac{2\cdot(Q\cdot t+V_{SY})}{R}$	-	Active leaching surface at time t

Resulting differential equation:

$$\frac{dC}{dt}=\frac{2\cdot K}{R}\cdot(C_S-C_0)-\left(\frac{2\cdot K}{R}+\frac{1}{t+V_{SY}/Q}\right)\cdot(C(t)-C_0)$$

General solution:

$$C_{(1)}(t)=C_0+D\cdot\frac{\exp\left(-\dfrac{2\cdot K\cdot(t+V_{SY}/Q)}{R}\right)}{t+V_{SY}/Q}+(C_S C_0)\cdot\left(1-\frac{R}{2\cdot K\cdot(t+V_{SY}/Q)}\right)$$

The constant of integration D is determined by the initial condition $C(t=0)=C_S$:

Final equation for mineralization development during product withdrawal phase:

$$\boxed{C_{(1)}(t,t\le V_{PS}/Q)=C_S-(C_S-C_0)\cdot\frac{R}{2\cdot K\cdot(t+V_{SY}/Q)}\cdot\left(1-\exp\left(-\frac{2\cdot K\cdot t}{R}\right)\right)}$$

Radius increment during product withdrawal phase.

$$\frac{dm_{salt}}{dt}=\frac{1}{\rho_{salt}}\cdot\frac{dV_{salt}}{dt}=\frac{A}{\rho_{salt}}\cdot\frac{dR_{salt}}{dt}=K\cdot A\cdot\big(C_S-C_{(1)}(t)\big)$$

$$\frac{dR}{dt}=\frac{K}{\rho_{salt}}\cdot\big(C_S-C_{(1)}(t)\big)$$

In order to determine the radius increase ΔR(h) in height h (h is counted from the starting product-brine level in upward direction), the differential formula has to be integrated for the time span the product-brine interface needs to move from height h to the total cavern height H.

$$R_1(h)=\frac{K}{\rho_{salt}}\cdot\int_{t_0=\frac{\pi\cdot R^2\cdot h}{Q}}^{t_1=\frac{\pi\cdot R^2\cdot H}{Q}}\left(C_S-C_{(1)}(t)\right)dt=\frac{K}{\rho_{salt}}\cdot\int_{t_0=\frac{\pi\cdot R^2\cdot h}{Q}}^{t_1=\frac{\pi\cdot R^2\cdot H}{Q}}\frac{(C_S-C_0)\cdot R}{2\cdot K\cdot(t+V_{SY}/Q)}\cdot\left(1-\exp\left(-\frac{2\cdot K\cdot t}{R}\right)\right)dt$$

Attachment 3 – Calculation of the saturation phase

In this phase the brine-filled volume and the active dissolution surface are constant.

$$C(t+\Delta t)\cdot V=C(t)\cdot V+K\cdot A\cdot\left(C_S-C(t)\right)\cdot\Delta t$$

with

$C(t+\Delta t)\cdot V$	-	salt mass in the cavern at $t+\Delta t$
$C(t)\cdot V$	-	salt mass in the cavern at t
$K\cdot A\cdot\left(C_S-C(t)\right)\cdot\Delta t$	-	salt dissolved during Δt from the cavern surface with area $A=2\pi\cdot R^2\cdot H$

Resulting differential equation:

$$\frac{dC}{dt}=\frac{2\cdot K}{R}\cdot\left(C_S-C(t)\right)$$

General solution:

$$C_{(2)}(t)=C_S-B\cdot\exp\left(-\frac{2\cdot K\cdot t}{R}\right)$$

The constant of integration B can be determined from the condition that the mineralization at the end of the product withdrawal phase must be identical to the mineralization at the beginning of the brine saturation phase:

$$C_{(1)}\left(t=\frac{\pi\cdot R^2\cdot H}{Q}\right)=C_{(2)}\left(t=\frac{\pi\cdot R^2\cdot H}{Q}\right)$$

Final solution:

$$\boxed{C_{(2)}(t,t>V_{PS}/Q)=C_S-(C_S-C_0)\cdot\frac{Q\cdot R}{2\cdot K\cdot(V_{PS}+V_{SY})}\cdot\left(\exp\left(\frac{2\cdot K\cdot V_{PS}}{Q\cdot R}\right)-1\right)\cdot\exp\left(-\frac{2\cdot K\cdot t}{R}\right)}$$

Radius increment during saturation phase:

$$\Delta R_2=\frac{K}{\rho_{salt}}\cdot\int_{t_1=\frac{\pi\cdot R^2\cdot H}{Q}}^{\infty}\left(C_S-C_{(2)}(t)\right)dt$$

$$\boxed{\Delta R_2(t > t_1) = \frac{R \cdot (C_S - C_0)}{2 \cdot \rho_{salt}} \cdot \frac{Q \cdot R}{2 \cdot K \cdot (V_{PS} + V_{SY})} \cdot \left(1 - \exp\left(-\frac{2 \cdot K \cdot V_{PS}}{Q \cdot R}\right)\right)}$$

KEYWORDS

leaching, numerical simulation, product withdrawal, cavern shape development